Campaign Expenditures & Bids in an All-Pay Auction

A Finance-Augmented Median-Voter Model

Abstract

In order to augment the Median Voter Model, we should ask "what is the role of money in the election process?" Particularly, why do candidates expend, and what is their optimal expenditure-level strategy? I suggest that candidates expend funds to capture swing mass via advertising in an all-pay mechanism. Using the discrete nature of the election outcomes to view expenditures as bids in an all-pay auction yields intuitive and consistent predictions of campaign expenditure levels, and election outcomes.

The model predicts continuous mixed-Nash campaign expenditure equilibrium strategies by the parties. The paper demonstrates rough validity of the model via simulation for an array of parameter values. Further, I assert a novel empirical method for generating efficient parameter estimates in models predicting continuous mixed-Nash equilibrium strategies. Last, with my parameter estimates, I analyze the counterfactual effect of different campaign finance reform policies on campaign expenditures and election outcomes.

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Introduction

In the 2004 Congressional Election, nearly $1 Billion was spent by parties' candidates trying to capture 2-year seats in the House of Representatives. The Median Voter Model, the standard model for policy determination and election outcomes, does not explain the role of finances in the election process. Essentially, the Median Voter Model is a 1st moment model of election outcomes; it only depends on the median of the relevant demographic distribution. By including campaign finance, I am able to introduce dependence on higher order moments of the district income distribution, and generate more consistent predictions of election outcomes and expenditure levels.

The purpose of this paper is to explain why and how parties expend campaign funds. I develop and estimate a model where party expenditures are bids in an all-pay auction. I assert that the election process is well described by an Asymmetric All-Pay Auction:

Auction - because the outcome is a discrete set

All-pay - because losing advertising expenditures are sunk

Asymmetric - because district demographics can yield a party advantage.

The model can then be used to provide structural counterfactual analysis for different campaign finance regulation policy changes.

Since the 1980s there have been several landmark breakthroughs in pursuit of explaining political expenditures on policy outcomes. Foremost, Becker (1983, QJE) reformulated the abundance of questions previously interested in understanding the influence on policy by competing groups. Also, in the mid-1980s, Hendricks and Porter developed a method to test empirically the continuous mixed-Nash predictions of asymmetric first price auctions with federal oil tract auction data. Finally, Amann and Leininger (GEB,1996) proved the existence, uniqueness, and calculate the continuous mixed-Nash equilibrium strategies to a first-price asymmetric all-pay auction.

With Becker's framed question, Amann and Leininger's theoretical tools, and Hendricks and Porter's empirical techniques, the timing is right for cultivating a finance-augmented median-voter model of strategic political expenditures.

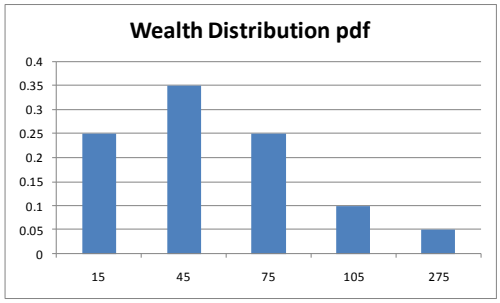
Parties' represent different positions along the single dimensional state space on the "role of the government." That is, Party1 represents the position that the government should collect funds for public goods only through efficient lump-sum taxation. The Party0 platform represents the position that the role of the government is to both collect funds for public goods and progressively redistribute income from the wealthy to the poor. The district demographics (income distribution and incumbency) and the ability of individuals to contribute funds to their preferred party generate asymmetries in party valuations and budgets; one party may have a multiplicative bid advantage over the other. Given the environment, I calculate the continuous mixed-Nash equilibrium expenditure strategy for each party.

There is no standard empirical method for testing continuous mixed-Nash. I first run a simulation for an array of parameter values, and compare the distributions of winner and loser expenditure levels for each case with data from the 2004 Congressional election. To officially estimate parameter values, some papers attempt to purify the equilibrium by adding in incomplete information. Others select only to test a couple moments of the predicted behavior function. I propose an M-estimation technique for attaining the parameter estimates in the continuous mixed Nash Equilibrium. The objective is to use the full set of information about the model's predicted equilibrium strategy, but also avail flexibility for distributional assumptions on error terms as per GMM.

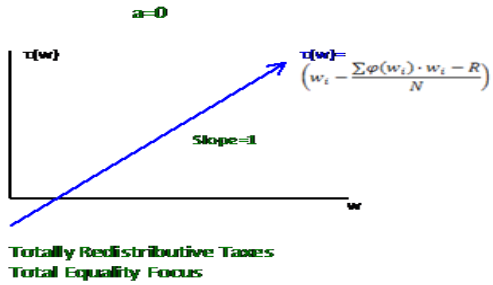
The contributions of this paper are to synthesize the question posed by Becker with the recent theoretical and empirical auction analysis techniques of Amann, Leininger, Hendricks, and Porter to construct and estimate a model of campaign expenditures as bids in a First-Price Asymmetric All-Pay Auction, FPAAPA.

Model

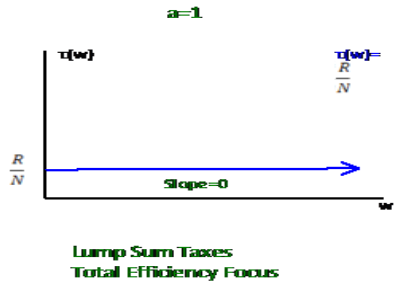
The environment is an election year. Every district, d, has voter income distribution; $\varphi_d(\omega)$. Two parties present vastly different platforms, generally represented by their proposed tax schedule. Party0 believes the role of government is to collect funds for the exogenous government revenue requirement, and also to redistribute income from the rich to the poor: resulting in an after-tax income distribution of singularity at mean income. Party1, on the other hand, offers lump sum taxes and places no importance on the redistributive role of government. Assume the public good procured with government revenue benefits all equally, so Party1's platform results in an after-tax income distribution equivalent to the pre-tax income distribution. All information is common knowledge.



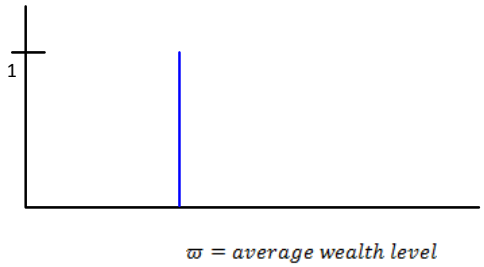
Party0 Tax Schedule



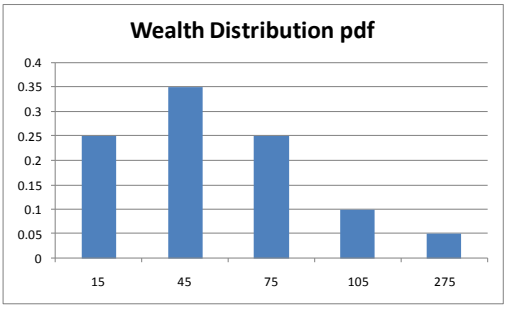
Party1 Tax Schedule



After Party0 Taxes



After Party1 Taxes



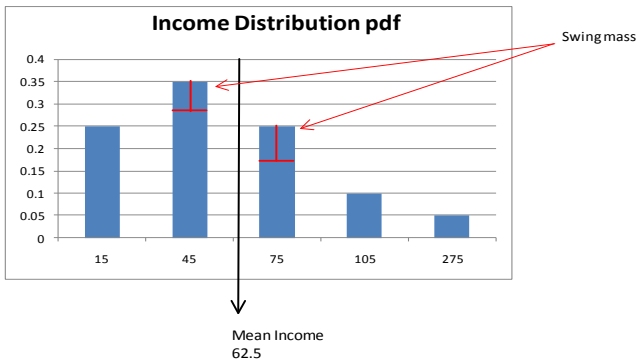
The standard Median Voter Result states that if the median of the income distribution is less than the mean, Party0 will have more voter mass favor, will win the election and redistribution will occur. Instead, I non-trivialize the Median Voter Model by allowing voters to make political contributions to support the party of their choice.

Some models non-trivialize the unrealistic median-voter results by adding platform differences in more dimensions. While it is true that voters most likely cast their vote in response to multiple issues, I argue that tax policy is a good platform-representative issue. Tax policy is well observed publicly and easily quantifiable, therefore generally subject to less backroom policy trades. If so desired, platforms differences across more dimensions could later be added.

In this simple setting, voters have clear preferences; high-income voters favor Party1's lump sum taxes of Party1, while low-income voters favor Party0's redistributive taxes. Moreover, voters at the extremes of the distribution have more at stake in the election than voters with mean income.

Voters near the mean income level are roughly indifferent between to the two platforms; both tax policies leave them with roughly equal amounts of after-tax income. If agents have any uncertainty in their location relative to the mean, or uncertainty for any reason about which party's policy will benefit them, then there exists a cost, c, associated with searching for the differences in the platforms. 'C' will generate a mass of indifferent voters centered around the mean. I denote this the swing mass, μ. Without advertising this mass is indifferent between the platforms, but intrusive search-free advertising is capable of shaping their voting preferences.

$$\mu = \int_{\bar{\omega}-\frac{c}{\bar{\omega}}}^{\bar{\omega}+\frac{c}{\bar{\omega}}} \varphi(\omega)d\omega$$



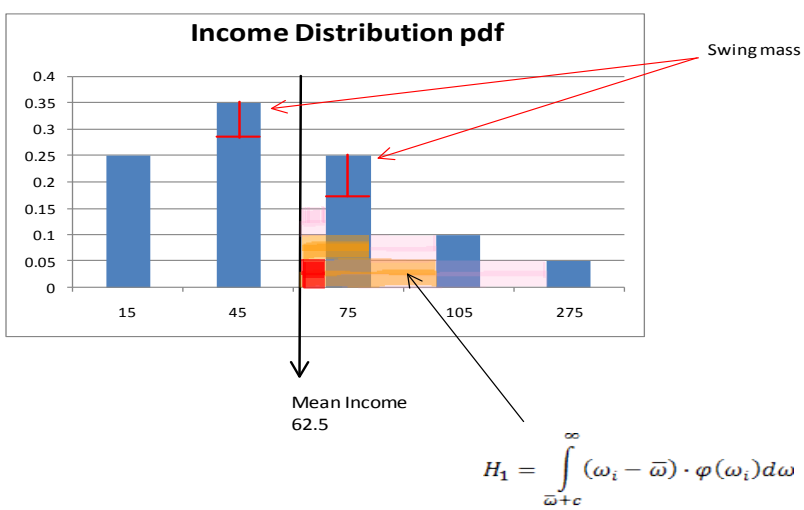
Voters at the extremes of the income distribution have more at stake. Knowing the existence of a swing mass responsive to intrusive advertising, income-extreme voters are thus willing to contribute funds to their party so it will have more funds available to expend on advertising. Assume in this simple environment that voters have risk-neutral preferences over wealth. Notice that their maximal contribution, in the case that they are pivotal is $\omega_i - \overline{\omega}$ if above mean income, and $\overline{\omega} - \omega_i$ if below.

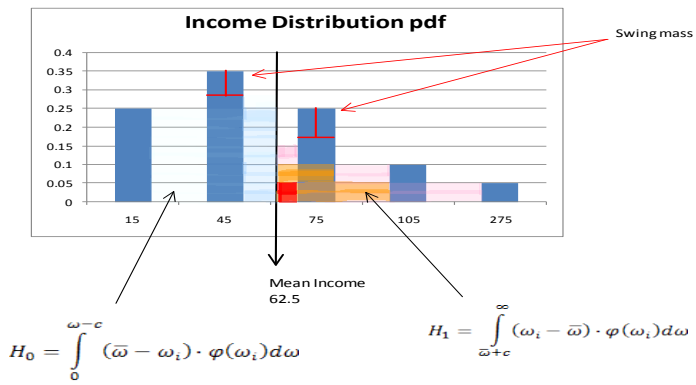
For a given income type i above mean income, the potential political contributions by this type are $\varphi_d(\omega_i) \cdot (\omega_i - \overline{\omega})$. For the highest income type, this value corresponds to the area in pink.

Similarly, for all voters favoring Party1, I compute their potential contributions. The sum of all these maximal contributions represents the potential budget for Party1: defined as

$$H_1 = \int_{\overline{\omega}+c}^{\infty} (\omega_i - \overline{\omega}) \cdot \varphi(\omega_i) d\omega$$

Notice that some colors represent area that must be counted twice, for different income types.





By similar token, I calculate the valuation of the office by Party0;

$$H_0 = \int_0^{\bar{\omega}-c} (\bar{\omega} - \omega_i) \cdot \varphi(\omega_i) d\omega.$$

Parties can be thought of as the aggregate of individuals desiring a particular policy platform since they receive their valuation for winning the elected office as the sum of all potential contributions. There are a couple of caveats, however.

Caveat	Explanation	Impact
Campaign Finance Regulation	Current Regulation consists of a cap on per capita contributions.	Since H1 is comprised more so of fewer contributions but at larger levels, H1 drop more than H0
Networking Technology	Until recently, it was difficult for parties to practically collect large numbers of small-scale contributions. The Obama campaign has successfully used the internet to make an advance in this category.	Previous networking failures would have H0 drop more than H1, but advances in networking technologies can be thought to lessen this impact.
Redistributive Distortion	Due to realistic distortion in labor supply arising from the moral hazard of over-redistribution, Party0 generally offers a tax schedule with slope<1	Low income types have less to gain. H0 drop
Liquidity Constraints	For very low income types, minimal liquidity prevents them from giving all they have, even in the case of being pivotal, to realize a better tax schedule in the future.	Low income types give less than stakes. H0 drop
Risk Aversion	Agents' uncertainty in lottery outcome may yield less contributions.	Ambiguous. Depends if crra, cara.
Free Riding	Becker (QJE 83) points out that party fund-raising efficiency is a major factor in party success.	Again, since Party0 contributions come from more, but smaller donations, the probability of each being pivotal is smaller, and harder to illicit. H0 drop
Mixing	District demographics (such as religion, education, and incumbency) and the other dimensions of platform preferences cause mixing, where low income types favor Party1 and high income types favor Party0.	Ambiguous. Depends on district demographics.

Define Z to be the operator taking into consideration the above caveats. Z converts H into an effective budget/valuation, V.

$$V_0 = z_0(H_0) \qquad \& \qquad V_1 = z_1(H_1)$$

Parties are risk-neutral and strive to win their valuation for the lowest bid: x represents the bid /expenditure of Party0, and y of Party1. Assume parties win the fraction of swing mass corresponding to the fraction of their expenditure among total expenditure.

The Party0 Optimization Problem -

$$Party0: \quad max_x \quad V_0 \mathbb{I}_{win_0} - x \qquad st \quad x \leq V_0$$

$$\text{where:} \qquad \left(\frac{x}{x+y}\right) \cdot \mu + mass_0 > \left(\frac{y}{x+y}\right) \cdot \mu + mass_1 \quad \Rightarrow \; win_0$$

$$\text{equivalently:} \qquad x(\mu + mass_0 - mass_1) > y\big(\mu - (mass_0 - mass_1)\big) \; \Rightarrow \; win_0$$

For notational simplicity, I redefine$(mass_0 - mass_1)$, the difference between mass of voters favoring Party0 and the mass for Party1 before the swing mass game, as b. $b = mass_0 - mass_1$

$$\text{Party0:} \quad max_x \; V_0 \cdot G_1\left(x \cdot \left(\frac{\mu+b}{\mu-b}\right)\right) - x \qquad st \quad x \leq V_0$$

$$\text{Party1:} \quad max_y \; V_1 \cdot G_0\left(y \cdot \left(\frac{\mu-b}{\mu+b}\right)\right) - y \qquad st \quad y \leq V_1$$

Since the payoff must be a constant for all strategies being mixed among.

$$\forall \, (x^*, y^*) \in X^* \cdot Y^*, \qquad \big(\pi_0(x^*), \pi_1(y^*)\big) = (k_0, k_1)$$

$$\text{For Party0:} \quad V_0 \cdot G_1\left(x \cdot \left(\frac{\mu+b}{\mu-b}\right)\right) - x = k_0 \qquad \Rightarrow \qquad G_1(y) = \frac{k_0 + y \cdot \left(\frac{\mu-b}{\mu+b}\right)}{V_0}$$

$$\text{For Party1:} \quad V_1 \cdot G_0\left(y \cdot \left(\frac{\mu-b}{\mu+b}\right)\right) - y = k_1 \qquad \Rightarrow \qquad G_0(x) = \frac{k_1 + x \cdot \left(\frac{\mu+b}{\mu-b}\right)}{V_1}$$

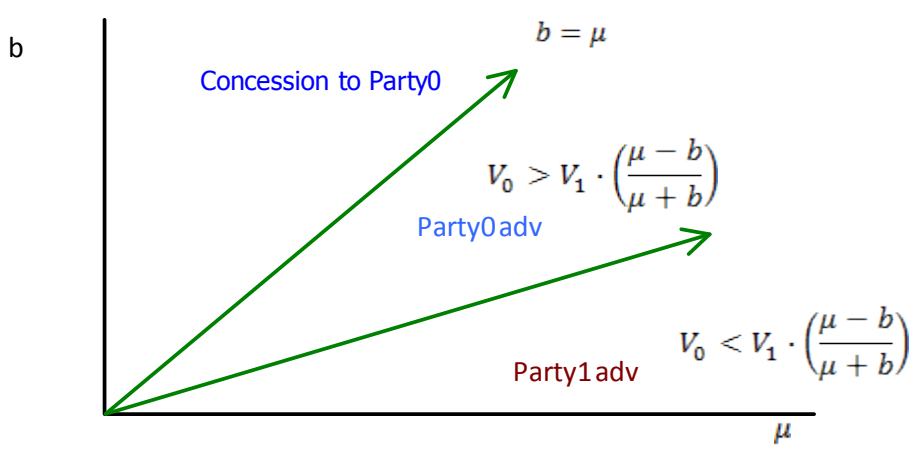
It is worthwhile to note that in the case that $b = 0$ & $V_0 = V_1$ we are in the standard symmetric first-price all-pay auction (FPAAPA) game. In fact there exist a 1-dimensional class of $b^{'}s \; and \; V^{'}s$ that all yield a symmetric game. If $b \neq 0$, I can still find suitable $V^{'}s$ such that neither Party is in an advantaged state. These are V's such that, if both parties expend their entire budget, the game ends in a draw. This result stems from the allowed asymmetry of valuations.

3 fundamental cases arise for the generic situation of b>0 (income distribution with median<mean).

Case	Conditions	Equilibrium Outcome
i concession to Party0	$b > \mu$	Neither Party expends Funds. The "mass advantage" is larger than the swing mass The outcome cannot be altered. x*=y*=0
ii Party0 Adv	$b < \mu$ $V_0 > V_1 \cdot (\frac{\mu - b}{\mu + b})$	Both Parties Expend Funds. Party0 retains advantage. Party1 gets 0 expected Profits Party0 expected to win and x*<V0
iii Party1 Adv	$b < \mu$ $V_0 < V_1 \cdot \left(\frac{\mu - b}{\mu + b}\right)$	Both Parties Expend Funds. Party1 has large enough "financial advantage," Takes Overall Advantage. Party0 gets 0 expected Profits Party1 expected to win and y*<V1

(Footnote: it is possible for b<0. Income distributions generically have median<mean, thus I focus on this case, but for large values of c (a large population vulnerable to advertising, it is possible, Party 1 has the mass advantage, b<1. The V's separately determine which party has "finance advantage," and therefore also the "overall advantage") Advantage analogous to info advantage (Hendricks AER 86).

District State Space

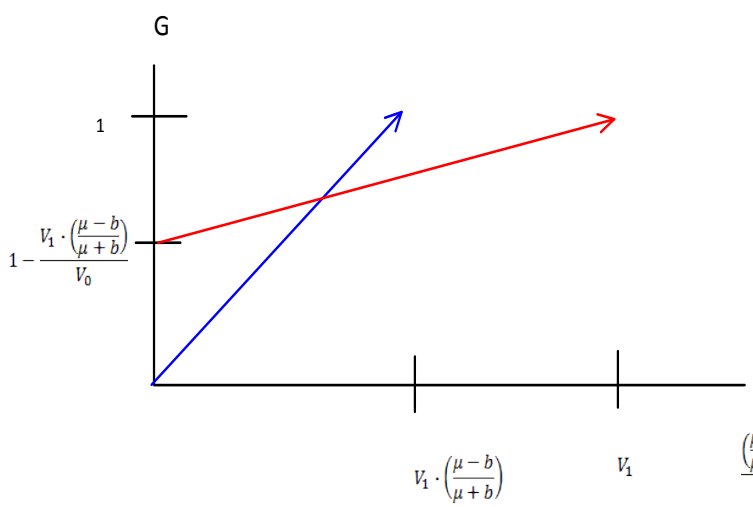


Results :

(Derivation using boundary conditions in Mathematical Appendix)

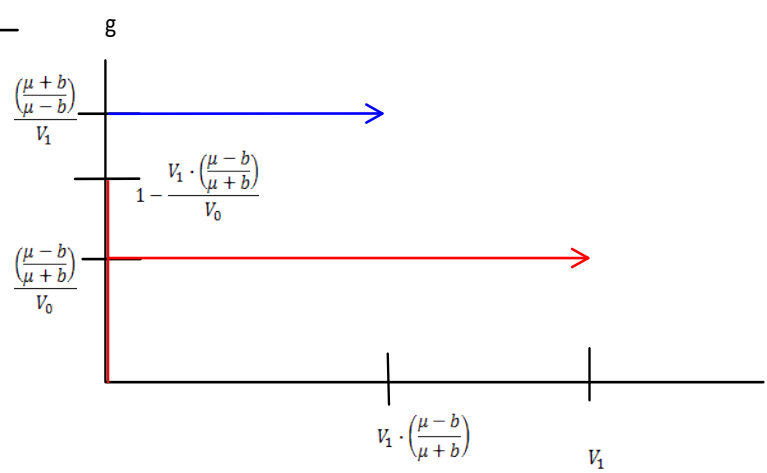
For case ii, the equilibrium expenditure strategy for party1: $G_1(y) = 1 - \frac{(V_1 - y)\cdot\left(\frac{\mu - b}{\mu + b}\right)}{V_0}$

And, the equilibrium expenditure strategy for party0: $G_0(x) = \frac{x\cdot\left(\frac{\mu + b}{\mu - b}\right)}{V_1}$



The graph depicts the Cdf's of the 2 parties' mixing equilibrium bidding strategies for a given environment; income distribution and parameter values such that in case ii.

Blue represents Party0 and Red represents Party1.



This graph depicts the pdf's of the parties mixed equilibrium bidding strategies.

Notice the mass point for Party1 at 0.

Case iii and cases with b<0 are trivially similar. The colors and the subscripts reverse.

The testable prediction of the model is a function. The standard functional hypothesis test is Kolmogorov-Smirnov. From the function, moments of the mixing behavior could also be tested for significance.

Since most district income distributions have a shape (roughly log normal) such that as the search cost of participation, c, increases (the bin size around the mean increases), the mass advantage trends to favor party 1: evidence of significance of 3rd moment left-skew in the district income distributions. The mass advantage trend also contributes to a financial advantage trend; higher relative amounts of contributors for Party1. Further, as c increases, the importance of the financial advantage grows as the mass of swing votes increases. Altogether, as c increases, Party0's mass advantage decreases, the swing mass increases, and the finance advantage increases (the case-determining lines rotate counter clockwise) all in favor of Party1. Party1 prefers an advertising susceptible populous, larger 'c' values.

As 'c' increases, party 1 wins dramatically more frequently and expends less money making parameter 'c' identifiable. In fact, the simulation suggests c=.3, implying that in an average district of 210,000 around 60,000 or 2/7ths of district votes determined by advertising.

The 2nd and 3rd moments of the district income distribution strongly influence the case of the district and thus expected expenditure levels and outcomes. We can begin to see why district drawing to form districts of particular demographics is critical among parties. Party1 desires a district with income distribution with high median/mean ratio, low variance, and large left skew: Party0 the opposite.

This result suggest that in federal level negotiations, to incur small expenditures from the costly all-pay mechanism for the next election, parties should trade district boundaries to make the district case very asymmetric. The model lends explanatory power to the gerrymandering process.

Counterfactual analysis:

Those who advocate an increase in the per capita contribution cap argue that limitations infringe on the 1st amendment right to free speech. Those who advocate tighter restrictions on the cap justify their position with the Federalist Paper, the philosophical underpinning of the Constitution. The Federalist Papers caution against the abuse of power by factions and advocate an immutable (and preferably equal) allocation of power in the election process.

The structure of the model allows analysis of campaign finance reform policies. In general, increases in the cap favor Party1. Depending on parameter values, Party0 usually has more individuals constrained, but each Party1 contributor is more constrained. As I increase the cap, I find that Party1 wins more frequently. Changes in the expenditure levels depend on the district demographics. In districts where Party0 previously held the advantage, the auction becomes more symmetric and expenditures increase for both parties. In districts where Party1 had the advantage, increases in the cap lead to more asymmetry and Party1 winning more frequently while expending less. Decreases in the cap generically favor Party0 and asymptotically yield the standard Median-Voter Result.

Data:

I have data from the 2004 Congressional Election; income distribution by district from the Census, and expenditure finances and outcomes by district from the FEC. Data snapshots in Data Appendix.

Empirics:

The parameters of my model consist of 'c', as well as any coefficients of regressors I use to control for the district-party valuation process (the Z operator mentioned at bottom of pg 7).

In particular, I just use the reduced-form valuation process given by

$$V_{0,d}\left(\Theta, \varepsilon_{\mathrm{d},0}\right) = \left(\beta_0 + \beta_{\mathrm{I}} \cdot \mathrm{r}_{\mathrm{I},0} + \varepsilon_{\mathrm{d},0}\right) \cdot \int_{0}^{\bar{\omega} - \frac{c}{\bar{\omega}}} \min\{\omega_i, \bar{\omega} - \omega_i, 5000\} \cdot \varphi(\omega_i) \mathrm{d}\omega_i$$

$$V_{1,d}\left(\Theta, \varepsilon_{\mathrm{d},1}\right) = \left(\beta_1 + \beta_{\mathrm{I}} \cdot \mathrm{r}_{\mathrm{I},1} + \varepsilon_{\mathrm{d},1}\right) \cdot \int_{\bar{\omega} + \frac{c}{\bar{\omega}}}^{\infty} \min\{\omega_i - \bar{\omega}, 5000\} \cdot \varphi(\omega_i) \mathrm{d}\omega_i$$

In light of the factors and effects outlined in the table on pg 7, I control for party of incumbency (dummy $\mathrm{r}_{\mathrm{I,p}}$, with common coefficient β_{I}) and differing party constants β_{p} representing cumulative effects of party free-riding problems, risk-aversion, networking problems, and redistributive distortion. In addition I control for a liquidity constraint (that poor don't go into debt to make political contributions) and a per capita contribution cap of $5k both accounted for within the argument of the integral.

Altogether, the parameter for this specification of the model are represented by vector Θ.

$\Theta = \{\mathrm{c}, \beta_0, \beta_1, \beta_{\mathrm{I}}\}$ in addition to any distributional assumption parameters for ε

Method 1: Simulation over an array of parameter values.

Parameter	Explanation	Simulation Grid Lower Bound	Simulation Grid Upper Bound	Preferred Estimate
Cr	Cost Ratio of Income Thresholds for Advertising Influence	.1	1	.35
B0	Party0 Mass Scaling Factor	.1	1	.6
B1	Party1 Mass Scaling Factor	.1	1	.7
BI	Incumbent Party Mass Scaling Factor	.8	2	1.6

Method 2: M-Estimation of Parameter Values.

Theoretical outline of methodology explained in Empirical Appendix. Asymptotics await the summer.

Significance Tests:

Method A: Kolmogorov-Smirnov test of significance of functional predictions.

Method B: Extract moments of functional predictions and test significance of correlation (Hendricks Porter, 1988). Moments= mass pt size, support, mean, variance, and probability of winning all as a function of rho (degrees from symmetry).

Conclusion:

The role of money is striking in the election process. The predictions from augmenting the Median-Voter Model with campaign financing recover many interesting features of election expenditure and outcome data. In particular, assuming parties expend funds to attract swing mass through advertising appears reasonable and leads to valuations and expenditure strategies consistent with the data. The simulation parameter estimate for the magnitude of swing mass is sizeable. The returns to advertising therefore are large, explain the $1Billion expenditure level all while capturing the salient shapes of the bid distribution data.

Weaknesses of the model include assumptions that all voters vote and exogenously locating the parties at the extremes. Renormalization of the district mass can treat the voter participation problem. More difficult however is endogenizing Party location. In a static game with Endogenous location, both parties would choose to collocate at the pivotal location in the augmented game. To prevent collocation in the Hotelling game, which is rarely observed (Parties favor different tax schedules), I propose to extend the model with dynamics and party platform adjustment costs.

Strengths of the model are its structural simplicity, predictive accuracy, and explanatory power beyond the questions posited with insight into gerrymandering objectives.

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Mathematical Appendix

A1: Applying boundary conditions to obtain closed form bid strategies.

For case ii,

I use a boundary condition on $G_1(y)$ to identify k_0

Since, $y_{max} = V_1 \quad \Rightarrow \quad G_1(V_1) = 1 = \frac{k_0 + V_1 \cdot \left(\frac{\mu - b}{\mu + b}\right)}{V_0}$

And by recognizing Party1's outside option of 0 expenditures $G_1(0) = 1 - \frac{V_1 \cdot \left(\frac{\mu - b}{\mu + b}\right)}{V_0}$

Party0's boundary condition G=1 identifies the upper bound of Party0's support.

$$G_0(\bar{x}) = 1 = \frac{\bar{x}}{V_1} \cdot \left(\frac{\mu + b}{\mu - b}\right) \Rightarrow \bar{x} = V_1 \cdot \left(\frac{\mu - b}{\mu + b}\right)$$

Data Appendix

A2: Data Explanation

District Level Data for Income Distribution

2004 Income Distribution by District	Congressional District (at Large) (109th Congress), Alaska	Congressional District 3 (109th Congress), Arizona	Congressiona District 4 (109t Congress), Arizc
Total:	228,358	262,282	215,
Less than $10,000	9,104	18,924	21,
$10,000 to $14,999	10,220	14,404	17,
$15,000 to $19,999	8,733	13,042	18,
$20,000 to $24,999	12,643	18,069	24,
$25,000 to $29,999	10,099	11,779	20,
$30,000 to $34,999	11,463	15,697	16,
$35,000 to $39,999	11,111	13,703	13,
$40,000 to $44,999	11,904	16,834	12,
$45,000 to $49,999	12,109	14,639	9,
$50,000 to $59,999	21,909	17,679	17,
$60,000 to $74,999	28,163	25,444	18,
$75,000 to $99,999	33,893	34,023	14,
$100,000 to $124,999	21,959	18,499	4,
$125,000 to $149,999	10,807	9,500	2,
$150,000 to $199,999	8,552	8,778	1,

District Level Data on Party of Incumbency, Party Expenditures, and Election Outcomes

HOUSE GENERAL ELECTION CANDIDATES FINANCIAL ACTIVITY JANUARY 1, 2003 THROUGH DECEMBER 31, 2004

	CANDIDATE	INC CHL OPN	PTY	W L	GEN PCT	NET RECEIPTS	INDIVIDUAL CONTRIB	PAC CONTRIB	CANDIDATE SUPPORT	NET DISBURSE	CASH ON HAND
ALABAMA											
1	BELK, JUDY MCCAIN	C	DEM	L	36	$442,450	$121,725	$69,450	$238,725	$442,141	$438
	BONNER, JOSIAH ROBIN	I	REP	W	63	$1,180,892	$778,293	$368,350	$0	$1,015,702	$178,261
2	JAMES, CHARLES DEAN	C	DEM	L	28	$1,320	$1,640	$0	$1,259	$1,320	$0
	EVERETT, TERRY	I	REP	W	71	$2,001,443	$439,265	$323,050	$530,173	$1,937,038	$281,717
3	FULLER, BILL	C	DEM	L	38	$280,237	$162,037	$96,500	$0	$240,774	$30,260
	ROGERS, MICHAEL	I	REP	W	61	$2,120,515	$1,192,778	$815,863	$0	$1,892,267	$246,299
4	COLE, CARL ALLEN III	C	DEM	L	25	$25,553	$23,303	$1,500	$0	$25,496	$56
	ADERHOLT, ROBERT B	I	REP	W	74	$748,910	$405,913	$275,907	$0	$735,352	$168,519
5	CRAMER, ROBERT EDWA	I	DEM	W	73	$820,378	$391,907	$394,043	$0	$588,838	$1,325,123
	WALLACE, GERALD C	C	REP	L	26	$12,644	$10,994	$1,000	$500	$12,610	$33
6	BACHUS, SPENCER T	I	REP	W	99	$1,647,636	$637,879	$871,809	$0	$1,376,103	$867,656
7	DAVIS, ARTUR GENESTR	I	DEM	W	74	$1,086,622	$569,227	$498,757	$104	$1,061,606	$80,623
ALASKA											
0	YOUNG, DON E	I	REP	W	71	$2,482,929	$1,491,129	$894,328	$0	$1,747,897	$1,898,567
ARIZONA											
1	BABBITT, PAUL J JR	C	DEM	L	36	$1,298,530	$866,850	$367,740	$0	$1,274,852	$23,677
	RENZI, RICHARD GEORG	I	REP	W	58	$2,349,548	$838,848	$1,185,206	$0	$2,207,249	$140,677
2	CAMACHO, RANDY ROBE	C	DEM	L	38	$93,253	$46,473	$56,500	$0	$101,998	$2,124
	FRANKS, TRENT	I	REP	W	58	$804,990	$399,495	$353,394	$24,000	$738,525	$72,992

Empirical Appendix

Method 2: M-Estimation of parameter values

Estimation of models predicting function outcomes, such as mixed Nash equilibrium strategies, have historically presented serious complications. Early econometric (and thermodynamic) treatments of functional predictions generally attack the problem with maximum likelihood (entropy) estimation on a couple of interesting moments of the predicted equilibrium strategies. More recent methods, dubbed GMEE Generalized Maximum Entropy Estimation within the umbrella of M Estimation, attack the problem similarly, but with less restrictive assumption on the distribution of the error term. My motivation for wanting to apply this technique is that simply estimating a few interesting moments of the predicted equilibrium strategies (such as supports, means, variances, and probabilities of winning) can leave a lot of the model's valuable predictions on the table.

The objective function is suggestive of standard maximum likelihood estimation since it is the multiplicative sum of the product of probabilities of the observations. However, the assumption on the error term need not be complete.

$$max_{\Theta} \prod_{d=1}^{435} g_{0,d}(\Theta, \varepsilon) \cdot g_{1,d}(\Theta, \varepsilon) \quad st\ \varepsilon \sim \{moment\ restrictions\}\ \ (i)$$

For example, if it were assumed $E(\varepsilon) = 0$, the estimation problem becomes

$$max_{\Theta} \prod_{d=1}^{435} g_{0,d}(\Theta, \varepsilon_{\mathrm{d}}) \cdot g_{1,d}(\Theta, \varepsilon_{\mathrm{d}}) \quad st\ \frac{1}{435} \sum_{d=1}^{435} \varepsilon_d = 0\ \ (ii)$$

The error term allows for disturbances on my ad-hoc reduced-form assumption of party valuations.

$$V_{0,d}(\Theta,\varepsilon_{\mathrm{d}}) = \left(\beta_0 + \beta_{\mathrm{I}} \cdot \mathrm{r}_{\mathrm{I},0} + \varepsilon_{\mathrm{d},0}\right) \cdot \int_{0}^{\bar{\omega}-\frac{c}{\bar{\omega}}} \min\{\omega_i, \bar{\omega} - \omega_i\,, 5000\} \cdot \varphi(\omega_i)\mathrm{d}\omega_i$$

$$V_{1,d}(\Theta) = (\beta_1 + \beta_{\mathrm{I}} \cdot \mathrm{r}_{\mathrm{I},1} + \varepsilon_{\mathrm{d},1}) \cdot \int_{\bar{\omega}+\frac{c}{\bar{\omega}}}^{\infty} \min\{\omega_i - \bar{\omega}, 5000\} \cdot \varphi(\omega_i)\mathrm{d}\omega_i$$

$\Theta = \{\mathrm{c}, \beta_0, \beta_1, \beta_{\mathrm{I}}\}$ in addition to any distributional assumption parameters for ε

The strengths of the technique are that it uses the full amount of information about the predicted strategies and avails all the flexibility about distributional assumptions on the error terms as per GMM. Moreover, method easily allows for predicted strategies with discontinuous state spaces. The discontinuity of the state space is incorporated into defining the strategy pdf.

$$max_{\Theta} \prod_{d=1}^{435} g_{0,d}(\Theta,\varepsilon_{\mathrm{d}}) \cdot g_{1,d}(\Theta,\varepsilon_{\mathrm{d}}) \cdot f(\varepsilon) \quad st\ \varepsilon \sim N(0,\sigma^2)$$

Where $g_{0,d}(\Theta,\varepsilon_{\mathrm{d}}) = \frac{1}{V_1(\Theta,\varepsilon_{\mathrm{d}})} \cdot \left(\frac{\mu(\Theta)+\mathrm{b}(\Theta)}{\mu(\Theta)-\mathrm{b}(\Theta)}\right)$ if $x_d \epsilon \left[0, V_1(\Theta,\varepsilon_{\mathrm{d}}) \cdot \left(\frac{\mu(\Theta)-\mathrm{b}(\Theta)}{\mu(\Theta)+\mathrm{b}(\Theta)}\right)\right]$ & *case ii*

$\frac{1}{V_1(\Theta,\varepsilon_{\mathrm{d}})} \cdot \left(\frac{\mu(\Theta)-\mathrm{b}(\Theta)}{\mu(\Theta)+\mathrm{b}(\Theta)}\right)$ if $x_d \epsilon (0, V_0(\Theta,\varepsilon_{\mathrm{d}})]$ & case iii

$1 - [\frac{V_0(\Theta,\varepsilon_{\mathrm{d}})}{V_1(\Theta,\varepsilon_{\mathrm{d}})} \cdot \left(\frac{\mu(\Theta)-\mathrm{b}(\Theta)}{\mu(\Theta)+\mathrm{b}(\Theta)}\right)]$ if $x_d = 0$ & case iii

case ii if $\mathrm{b}(\Theta) < \mu(\Theta)$ & $V_0(\Theta,\varepsilon_{\mathrm{d}}) > \mathrm{V}_1(\Theta,\varepsilon_{\mathrm{d}}) \cdot \left(\frac{\mu(\Theta)-\mathrm{b}(\Theta)}{\mu(\Theta)+\mathrm{b}(\Theta)}\right)$

case iii if $\mathrm{b}(\Theta) < \mu(\Theta)$ & $V_0(\Theta,\varepsilon_{\mathrm{d}}) < \mathrm{V}_1(\Theta,\varepsilon_{\mathrm{d}}) \cdot \left(\frac{\mu(\Theta)-\mathrm{b}(\Theta)}{\mu(\Theta)+\mathrm{b}(\Theta)}\right)$

0 otherwise.

Similar procedure for $g_{1,d}(\Theta,\varepsilon_{\mathrm{d}})$.

Check for over-identifying restrictions, and asymptotics forthcoming this summer.

Simulation Appendix

A3-1: Parameter Values

Parameter	Explanation	Simulation Grid Lower Bound	Simulation Grid Upper Bound	Preferred Estimate
Cr	Cost Ratio of Income Thresholds for Advertising Influence	.1	1	.35
B0	Party0 Mass Scaling Factor	.1	1	.6
B1	Party1 Mass Scaling Factor	.1	1	.7
BI	Incumbent Party Mass Scaling Factor	.8	2	1.6

$$V_{0,d}(\Theta, \varepsilon_{d,0}) = (\beta_0 + \beta_I \cdot r_{I,0} + \varepsilon_{d,0}) \cdot \int_{0}^{\bar{\omega}-\frac{c}{\bar{\omega}}} \min\{\omega_i, \bar{\omega} - \omega_i, 5000\} \cdot \varphi(\omega_i) d\omega_i$$

$$V_{1,d}(\Theta, \varepsilon_{d,1}) = (\beta_1 + \beta_I \cdot r_{I,1} + \varepsilon_{d,1}) \cdot \int_{\bar{\omega}+\frac{c}{\bar{\omega}}}^{\infty} \min\{\omega_i - \bar{\omega}, 5000\} \cdot \varphi(\omega_i) d\omega_i$$

A3-2: Implied Variables from Preferred Parameter Estimates

<u>mu swing mass</u>: (districts) x (Cr values)

Swing Mass

	Cr values				
	0.1000	0.2775	0.4550	0.6325	
	25484.9	68257.5	113632.1	150878.3	1.0
	24887.7	68323.0	110868.8	148885.4	2.0
	18672.2	51782.0	83503.8	111508.0	3.0
	23071.2	66926.2	113656.6	155611.1	4.0
	25030.4	68295.8	112031.4	149655.9	5.0
	23715.3	69514.5	113857.3	156228.1	6.0 districts
	24945.4	68312.0	111339.3	149197.2	7.0
	23463.9	66586.7	113189.7	155412.9	8.0
	22652.9	67406.0	113742.2	155890.9	9.0
	23874.6	66231.6	112143.1	155205.6	10.0
	23511.7	69399.8	113837.3	156190.6	11.0
	23743.9	64725.9	108048.8	153221.6	12.0
	23484.8	66568.6	113136.4	155402.4	13.0
	23088.5	66911.2	113653.9	155602.3	14.0
	24237.5	67951.2	105805.5	142954.0	15.0
	…	…	…	…	…

‘b’ mass advantage=mass0-mass1=mass below median-mass above median

Mass Advantage

	Cr	values			
0.1	0.2775	0.455	0.6325		
42843.7	33171.4	20831.9	5031.1	1	
67493.4	56524.8	37790.2	10309.2	2	
51396.4	30063.6	17468.6	-9343.7	3	
60453.6	50015.6	31688.2	12984.9	4	
47502.9	33981.4	20808.8	10213.2	5	
62711.4	56476.4	42594.9	23184.4	6	districts
65667.5	47142.8	29177.0	16540.2	7	
54190.9	43322.6	33796.8	9135.4	8	
52665.5	45154.5	29215.6	13428.8	9	
46006.9	41791.0	33743.7	9170.5	10	
46934.9	40566.5	22200.3	8089.4	11	
50046.6	44333.6	31551.5	24011.5	12	
36828.8	33463.6	32451.9	10314.7	13	
28789.4	29864.5	26998.8	14555.2	14	
47744.3	44187.1	23328.9	6233.2	15	
…	…	…	…	…	

V's real budgets/valuations:

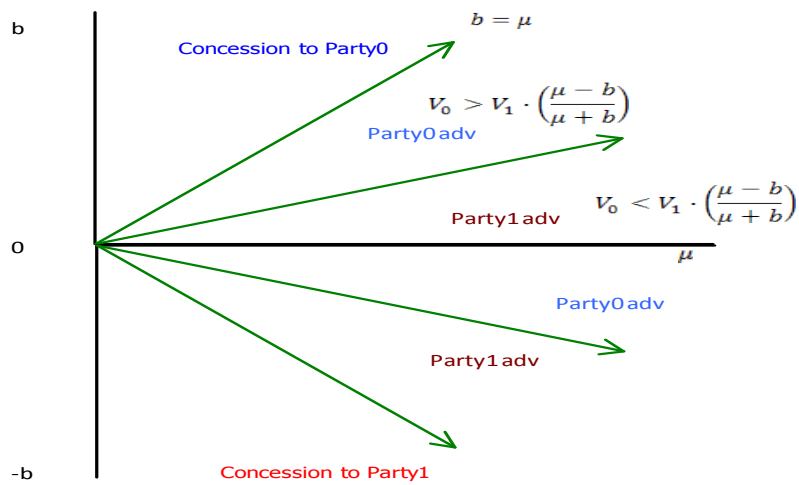
$$V_{0,d}(\Theta) = \left(\beta_0 + \beta_I \cdot r_{I,0}\right) \cdot \int_{0}^{\bar{\omega}-\frac{c}{\bar{\omega}}} \min\{\omega_i, \bar{\omega} - \omega_i, 5000\} \cdot \varphi(\omega_i) d\omega_i$$

$$V_{1,d}(\Theta) = (\beta_1 + \beta_I \cdot r_{I,1}) \cdot \int_{\bar{\omega}+\frac{c}{\bar{\omega}}}^{\infty} \min\{\omega_i - \bar{\omega}, 5000\} \cdot \varphi(\omega_i) d\omega_i$$

Cr=.35	BetaI=1.6	Beta0=.6	Beta1=.75

V0	V1		
1141564.2	8986057.9	1	
1347901.7	11703742.8	2	
2481387.3	1790049.3	3	
5842597.4	4545200.5	4	
3990459.5	2945857.1	5	
5613636.8	4962808.3	6	districts
4131046.8	3788890.1	7	
5826345.3	4312118.7	8	
1591423.7	13597956.2	9	
6066041.8	4101749.6	10	
4845104.7	3319595.5	11	
6186673.6	4685561.3	12	
5245458.2	3729562.2	13	
4700931.1	3238656.5	14	
1023979.6	9070004.4	15	
…	…	…	

Cases:



Labeled Clockwise:

0

1/8th

3/8th

5/8th

7/8th

1

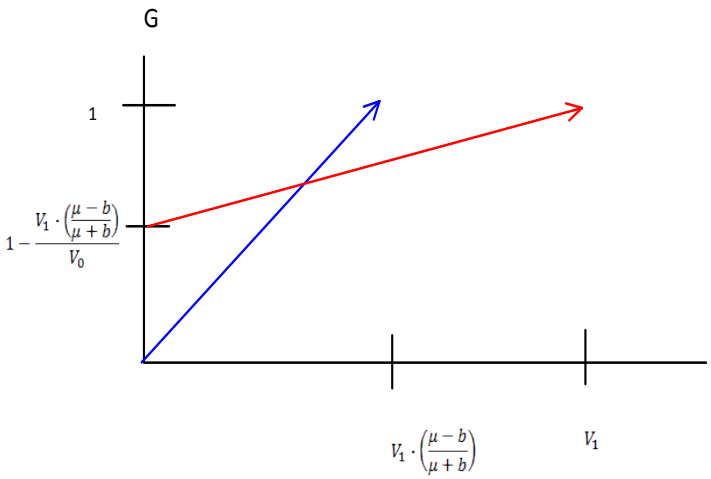
Beta1=1.6	Beta0=.6	Beta1=.75;

Cases

	Cr	values			
0.1	0.2775	0.455	0.6325		
0	0.375	0.375	0.375	1	
0	0.125	0.375	0.375	2	
0	0.125	0.125	0.875	3	
0	0.125	0.125	0.375	4	
0	0.125	0.125	0.375	5	
0	0.125	0.125	0.375	6	districts
0	0.125	0.125	0.375	7	
0	0.125	0.125	0.375	8	
0	0.375	0.375	0.375	9	
0	0.125	0.125	0.375	10	
0	0.125	0.125	0.375	11	
0	0.125	0.125	0.375	12	
0	0.125	0.125	0.375	13	
0	0.125	0.125	0.375	14	
0	0.375	0.375	0.375	15	
0	0.375	0.375	0.375	16	
0	0.125	0.125	0.375	17	
0	0.375	0.375	0.375	18	
0	0.375	0.375	0.375	19	
0	0.125	0.125	0.375	20	
0	0.125	0.125	0.375	21	
0	0.125	0.125	0.375	22	
0	0	0.125	0.375	23	
0	0.125	0.125	0.375	24	
0	0.125	0.125	0.875	25	
...	...	...	...	...	

Bid/Expenditure Strategies = G cdf, g pdf

For example, for a district in case 1/8th



Cr=.35	Beta1=1.6	Beta0=.6	Beta1=.75

Bid Strategies

	Party0		Party1		
0 mass	upper bound	0 mass	upper bound		
0.93	1141564.19	0.00	588049.89	1	
0.97	1347901.71	0.00	399854.50	2	
0.00	634385.02	0.74	1790049.26	3	
0.00	1495849.38	0.74	4545200.46	4	
0.00	1552919.35	0.61	2945857.06	5	
0.00	1378893.64	0.75	4962808.30	6	districts
0.00	1293426.33	0.69	3788890.09	7	
0.00	1559791.81	0.73	4312118.68	8	
0.95	1591423.73	0.00	628529.77	9	
0.00	1457801.72	0.76	4101749.65	10	
0.00	1489492.40	0.69	3319595.47	11	
0.00	1415039.59	0.77	4685561.29	12	
0.00	1745936.57	0.67	3729562.24	13	
0.00	1405180.94	0.70	3238656.52	14	
0.95	1023979.64	0.00	489108.98	15	
…	…	…	…	…	

Expenditure Levels = use random draw for each district and each party, and bid strategies

Random draws

random draws
~U[0,1]

Party0	Party1		
0.7477	0.0167	1	
0.6455	0.8009	2	
0.1232	0.1425	3	
0.5044	0.4785	4	
0.3473	0.2568	5	
0.0921	0.3691	6	districts
0.1478	0.6618	7	
0.1982	0.1696	8	
0.6723	0.2788	9	
0.4315	0.1982	10	
0.6944	0.1951	11	
0.2568	0.3268	12	
0.0098	0.8803	13	
0.5323	0.4711	14	
0.2794	0.404	15	
…	…	…	

Predicted Exp Levels

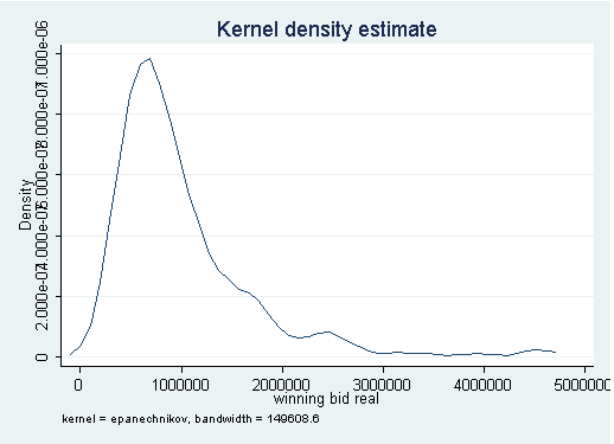
Party0	Party1		
0.00	392630.19	1	
0.00	259116.56	2	
22061.84	0.00	3	
438031.90	0.00	4	
1244574.38	415623.70	5	
477789.39	0.00	6	districts
107763.04	0.00	7	
797219.53	539392.80	8	
0.00	53029.35	9	
1078014.49	871256.34	10	
781596.49	0.00	11	
1138428.90	300477.39	12	
1426275.10	628657.40	13	
266241.71	0.00	14	
0.00	266997.84	15	
…	…	…	

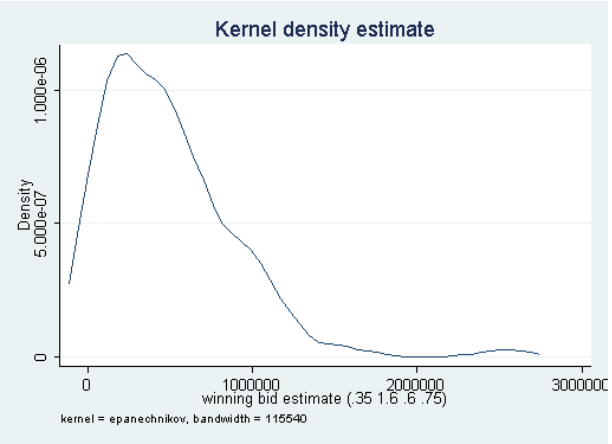
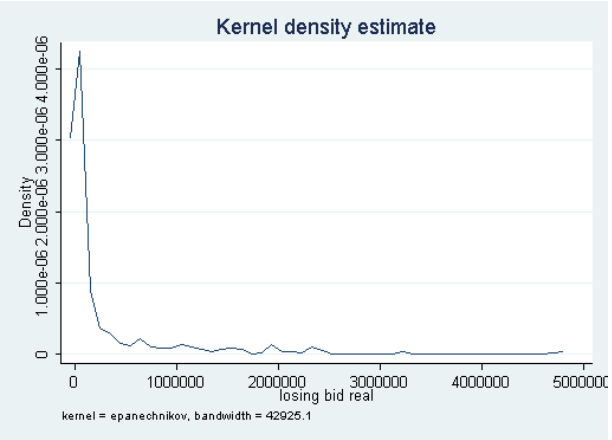
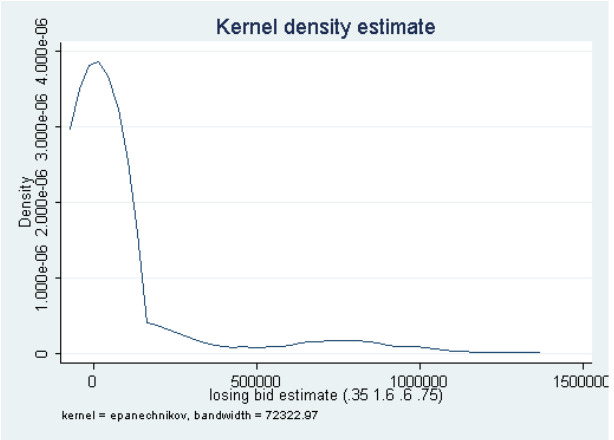
Expenditure Distributions:

Real Expenditures		Predicted Expenditures			
Winner	Loser	Winner	Loser		
1,747,897.00	6,309.00	291,088.10	0.00	1	
792,700.00	5,704.00	59,021.81	0.00	2	
624,271.00	478,502.00	268,272.30	0.00	3	
562,533.00	159,219.00	485,142.80	140,952.90	4	
571,957.00	154,677.00	929,459.00	0.00	5	
1,240,543.00	44,685.00	649,354.40	869,691.10	6	districts
783,143.00	31,883.00	900,159.20	37,002.79	7	
780,196.00	52,623.00	1,091,679.00	0.00	8	
1,018,433.00	84,998.00	512,501.40	0.00	9	
1,190,646.00	244.00	489,484.90	48,987.73	10	
455,735.00	148,631.00	983,708.20	102,486.20	11	
939,389.00	207,432.00	452,246.30	0.00	12	
539,475.00	23,341.00	926,855.40	739,519.60	13	
598,739.00	32,611.00	919,614.60	0.00	14	
667,520.00	605,280.00	484,193.50	0.00	15	
1,493,678.00	262,130.00	229,334.60	0.00	16	
1,009,290.00	3,540.00	1,109,403.00	0.00	17	
551,059.00	68,635.00	459,020.70	26,971.15	18	
1,338,730.00	790,869.00	136,220.30	0.00	19	
871,672.00	40,391.00	443,909.10	0.00	20	
902,390.00	6,196.00	509,261.30	118,222.00	21	
955,782.00	262,288.00	667,507.30	0.00	22	
453,715.00	85,456.00	902,587.90	77,886.48	23	
623,023.00	356,372.00	273,714.90	49,019.49	24	
527,054.00	1,994.00	97,811.42	0.00	25	
259,663.00	69,035.00	271,916.90	0.00	26	
572,055.00	212,154.00	95,555.47	0.00	27	
330,980.00	14,828.00	332,419.80	0.00	28	
765,112.00	68,081.00	599,870.90	0.00	29	
307,056.00	6,015.00	555,923.80	128,039.90	30	
273,757.00	16,968.00	672,906.90	528,890.90	31	
782,521.00	1,125,677.00	1,535,690.00	1,296,859.00	32	
…	…	…	…	…	

The simulation parameter estimates hold much room for refinement, but the general shapes and magnitudes of the predictions are quite modest.

Real vs. Predicted Winning Expenditures





Real vs. Predicted Losing Expenditures