

Testing the Permanent Income Hypothesis with Synthetic Cohorts  
Heterogeneous Discounting and Micro-Macro Data

Motivation

If it were found conclusive that the permanent income hypothesis were true (that individuals effectively smooth out their income inequalities in their consumption choices), then there would be little justification for government intervention to assist individuals in their savings. This conclusion would suggest that the need for Social Security programs (if their sole goal was to provide savings assistance) has been overemphasized and, normatively, should be scaled back.

Positive Question

What are individuals' intertemporal elasticity of substitution? What are individuals' time preferences ( $\beta$ ) and do they depend on demographics? Overall, are the estimated coefficients significant and does the model do a good job of explaining lifetime consumption behavior? Is there significant evidence that agents smooth their consumption in response to lifetime permanent income? Or, do agents' consumption levels fluctuate with variations in income levels across their lifetime? If so, what accounts for the apparent irrationality? If not, or inconclusive, what could have allowed previous economists to reject the permanent income hypothesis and thus continue to justify large government intervention programs such as Social Security?

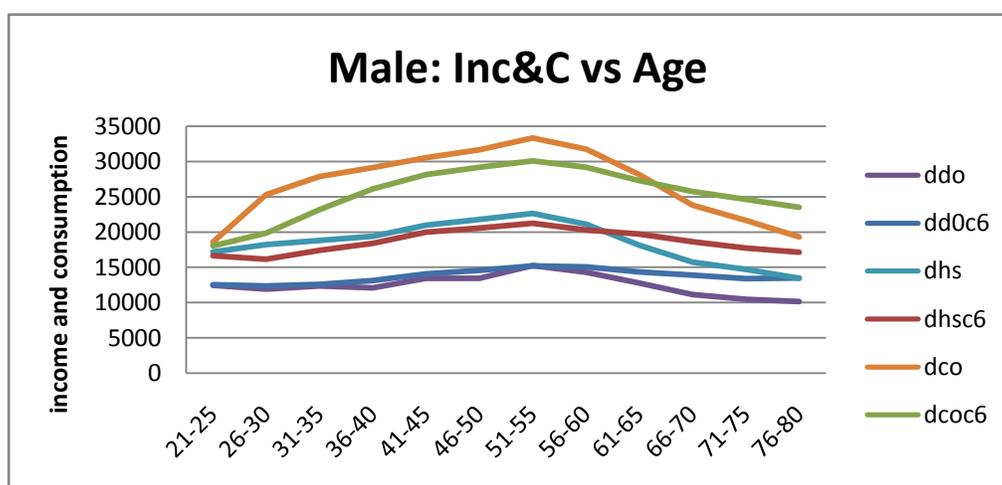
Methodology Overview

I follow the methodology of Attanasio (1995) for generating data on synthetic cohorts from micro data and for imposing structural utility preferences. I diverge from his procedure where he makes restrictive assumptions about distributions of the error terms to simplify the linearization and estimation. Instead, I take his resulting synthetic cohort data and nonlinearly estimate the structural utility parameters via GMM as in Hansen Singleton (83) for each cohort.

## Data

Ideal data for this analysis would consist of consumption, income levels, and savings rates faced by agents for every period throughout their lifetime. Instead, my data consists of 20 years of annual cross-sectional micro-level data from the March CEX (1980-2000). I take the data, and divide the individuals into 6 different cohorts based on gender and educational attainment (drop out, high school, college). For each cohort, I observe each individual's age, consumption level, income level, and interest rate faced. Thus I construct a balanced panel age profile of income, consumption, and interest rates for each cohort.

I use this constructed Micro-Level Macro-Aggregated Data to estimate the structural utility parameters for each cohort.



The interest rate age profiles for each cohort are in the appendix. In addition, all data (original), synthesized panel data, and estimates are contained in Excel Format on my website: <http://mastresearch.googlepages.com/> under the topic 'EconometricsIII'.

One drawback the Synthetic Panel Data set is that the consumption and income data was at the household level and therefore, ad hoc assumptions about the public good aspect and intra-household sharing rules must be asserted in order to achieve data on individual level consumption and income levels. I asserted that the sharing rule was that within a household, each family member fully enjoyed all energy and housing consumption, and that the rest of the expenditures were on 60% public goods (along a linear homotopy where 0 represents equal per capita consumption with no sharing and 1 represents pure public good where each member enjoys equally full consumption of the good). The following analysis and estimates did not change dramatically for variations in the sharing rule.

## Methodology

Dynamic Optimization (for each cohort)

$$\max_{\{c_t, s_t\}_{t=1}^{12}} E_0 \sum_{t=1}^{12} \beta^t U(c_t) \text{ st } c_t + s_t \leq y_t + (1 + r_t) \cdot s_{t-1} \quad \forall t$$

Where  $U(c_t) = \frac{c_t^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}}$  and  $s_{-1} = 0$  and  $s_{12} = 0$ .

Euler Equation

$$\forall t \quad E[\beta \cdot (1 + r_t) \cdot \frac{U'(c_t)}{U'(c_{t+1})} - 1] = 0$$

The individuals' residuals should be mean 0.

Euler Equation to be estimated

$$\forall t \quad E[\beta \cdot (1 + r_t) \cdot \left(\frac{c_{t+1}}{c_t}\right)^{\frac{-1}{\sigma}} - 1 | Z] = 0$$

The econometrician's residuals (the correlation of the individuals' residuals and elements from the individuals' information set) should be mean 0.

Parameters and Instruments

$$\beta = \theta_0 + \theta_1 \cdot \text{age} + \theta_2 \cdot \text{age}^2 + \theta_3 \cdot (\text{family size})$$

$$\theta = \{\sigma, \theta_0, \theta_1, \theta_2, \theta_3\} \text{ for just identified}$$

$$\theta = \{\sigma, \theta_0, \theta_1, \theta_3\} \text{ for over identified case}$$

$$Z = \{\text{current income, c lag 1, c lag 2, current r, r lag 1}\}$$

## Results

### Parameter Estimates

just identified			5 parameters				
5 instruments			sigma	beta const	beta age	beta age2	beta fam siz
income		male do	0.7285	0.8116	-0.0124	0.0002	0.0658
consumption		male hs	0.51	0.9543	-0.026	0.0003	0.0909
lag consumption	cohort	male co	0.5517	0.8305	-0.0307	0.0004	0.1329
interest rate		fe do	0.8316	0.7983	0.0035	0	0.0241
lag interest rate		fe hs	0.8467	0.7993	-0.0015	0.0001	0.024
		fe co	1.211	0.4466	0.0248	-0.0002	-0.0575
			2 parameters				
2 instruments			sigma	beta			
income		male do	0.7304	0.8964			
interest rate	cohort	male hs	0.7224	0.8925			
		male co	0.725	0.8911			
		fe do	0.7121	0.902			
		fe hs	0.7236	0.8954			
		fe co	0.7337	0.8898			
			4 parameters				
over identified			sigma	beta const	beta age	beta famsize	
5 instruments							
income		male do	1.2711	1.2684	-0.0013	-0.0696	
consumption		male hs	3.1781	-1.5978	0.0229	0.4953	
lag consumption	cohort	male co	2.2544	-0.5766	0.0138	0.3131	
interest rate		fe do	1.1786	1.4576	-0.0059	-0.0576	
lag interest rate		fe hs	8.1622	-0.5731	0.0156	0.2813	
		fe co	2.2723	1.3354	-0.0012	-0.105	

### Weighting Matrix

Very problematic, non-singular. 1<sup>st</sup> stage estimates (pseudo parameters) yield very small residuals. Thus, unable to invert, nor needed to invert, to get more efficient estimates.

### Variance Covariance Matrix

Without the weighting matrix, I am also unable to compute the Variance Covariance Matrix for the estimated parameter values, although I still present the results and the code.

## Conclusions

The permanent income hypothesis cannot be conclusively rejected. The results indicate that there is reason to believe that individuals are rational consumers with concave preferences and do effectively consumption smooth. The need for savings assistance programs is inconclusive at best. The drop in consumption levels at retirement is not disastrously severe, especially considered relative to drops in consumption levels arising from other income shocks, such as illness, loss of job, or going back to school.

Possible reasons for previous rejections of the permanent income hypothesis, and thus justifications for government intervention in individual savings, could come from the fact that discounting was neither considered heterogeneous across individuals nor age dependent. Dismissal of patience heterogeneity and time varying discounting would lead to noisier estimates on the IES. The results of this paper show that beta heterogeneity and age-variation are significant and critical for policy analysis.

## Synthetic Cohort Panel Data

income		age											
		21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70	71-75	76-80
	male do	12433.97	11885.37	12373.45	12082.99	13448.83	13422.52	15231.91	14299.54	12743.25	11157.76	10463.3	10147.98
cohort	male hs	17170.4	18229.63	18807.35	19416.28	20970.34	21787.27	22648.81	21129.62	18128.47	15710.65	14688.99	13426
	male co	18565.22	25275.66	27857.77	29112.41	30538.77	31626.27	33313.53	31739.93	28148.44	23810.7	21633.14	19279.96
	fe do	9462.289	9627.01	10058.33	11529.92	12337.83	12521.68	12738.15	11507.25	10373.41	9623.15	9760.527	8821.395
	fe hs	15665.4	16953.9	17387.77	18644.81	20616.24	21740.25	22091.82	18876.08	16074.01	14583.78	12741.51	11532.61
	fe co	19004.24	25974.66	27487.76	28427.11	30024.7	31149.51	31109.09	28876.76	23599.67	19222.79	17763.15	16602.37
consumption		age											
		21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70	71-75	76-80
	male do	12549.12	12371.23	12617.28	13132.75	14055.6	14554.76	15193.83	15080.17	14344.37	13897.74	13420.79	13487.56
cohort	male hs	16603.94	16121.09	17390.46	18375	19965.31	20555	21232.5	20295.05	19672.16	18597.24	17697.54	17129.1
	male co	18032.56	19811.2	23152.44	26110.84	28129	29164.82	30053.67	29172.05	27245.03	25711.36	24594.56	23470.71
	fe do	10785.24	11239.89	11897.73	13354.98	13538.05	13996.09	13880.05	13463.13	13039.09	12348.23	12178.61	11895.93
	fe hs	15563.29	16109.37	17577.28	19141.23	20717.59	20968.6	21076.07	19710.95	18792.65	17561.43	16549.91	16045.79
	fe co	17929.81	21030.83	23736.3	26193.56	27688.84	29322.13	27748.68	27291.66	24814.64	24050.99	21665.83	19776.64
interest rates		age											
		21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70	71-75	76-80
	male do	10.00297	9.72655	9.555598	9.71998	9.999866	10.1138	10.36058	10.39393	10.23784	10.16987	10.11266	10.01274
cohort	male hs	9.958752	9.818395	9.52562	9.34572	9.270262	9.328299	9.599019	9.810506	9.690509	9.459792	9.19574	8.972885
	male co	9.929363	9.804298	9.754455	9.37095	9.148766	9.152916	9.280869	9.349159	9.408258	9.200048	9.12119	9.299544
	fe do	10.05595	9.729675	9.708851	9.840633	9.935098	10.22019	10.33363	10.26399	10.23313	10.34761	10.05807	10.00215
	fe hs	10.01929	9.825171	9.532388	9.412213	9.312327	9.399898	9.560371	9.727843	9.66114	9.358014	9.246871	9.142971
	fe co	9.812435	9.635979	9.457424	9.134036	8.942924	8.923473	9.244067	9.357209	9.270244	9.40067	9.438761	9.288634

All original CEX data, synthetic panel data, interest rates, and estimates are located on my website: <http://mastroresearch.googlepages.com/> under the heading, 'Econometrics III'.

MatLab Programs:

MCP.m

(MetricsIII Cooper Project) (calls GMMObjFunId.m, GMMObjFun.m, and VarCov.m)

GMMObjFunId.m

(Generalized Method of Moments Objective Function) (calls VOM.m)

VOM.m

(Vector of Moments) (calls Euler.m)

Euler.m

(Euler Equations) (calls BETA.m)

BETA.m

(Beta discount parameter values)

GMMObjFun.m

(GMM Objective Function with Optimal weighting matrix)(calls VOM.m)

VarCov.m

(Variance Covariance Matrix)(calls JAC.m)

JAC.m

(Instrument Interacted Jacobian Matrix)(calls JACJ.m)

JACJ.m

(Jacobian Vector)(calls BETA.m)

## MCP.m

```
% Econometrics III
% Cooper Project
% Spring 2008, Apr 25
% Nick Mastronardi
% Data from CEX March (1980-2000)
% Program Name: HS Mod with Micro-Macro Data
% Program Name: Identifying Utility Parameters
% Program Name: Testing Permanent Income Hypothesis
% Synthetic Panel Data and Nonlinear GMM Estimation

clear all
clc

global c inc r Z ob age nsc age2 famsize owm

load data.mat
%set of data matrices

%r = 6x12 interest rates for each time (cohort type) x (time period)
%c = 6x12 consumption values for each (cohort type) x (time period)
%age, age2
%1x12 regressors for each time period (x6), same for all cohorts (x1)
%inc, famsize
%6x12 beta regressors for each (cohort type)x(timeperiod)

disp('Synthetic Panel Data and Nonlinear GMM Estimation')
disp('Identifying Utility Parameters')

scd=size(c);
sc=scd(1); %sc=#synthetic cohort types
ob=scd(2); %ob=#effective observations

%b=zeros(sc,5); %b=matrix of optimal parameter values for each cohort
b=zeros(sc,4);
%b=zeros(sc,2);

%vcb=zeros(5,5,sc); %vcb=varcov matrix of parameters for each cohort
vcb=zeros(4,4,sc);
%vcb=zeros(2,2,sc);

Qval=zeros(sc,1);

for nsc=1:sc %for each cohort

    %generate/extract instrument vector Z, items uncorr w Euler residu
    %9x5 (each usable time period lag)x(5 instruments)

    Z=zeros(ob-3,5);
    %Z=zeros(ob-3,4);
    %Z=zeros(ob-3,2);
```

```

    for nob=3:ob-1 %3 bc two lags, -1 because can't do euler on last
        Z(nob-2,:)=[inc(nsc,nob), c(nsc,nob-1), c(nsc,nob-2), r(nsc,nob),
r(nsc,nob-1)];
        %Z(nob-2,:)=[inc(nsc,nob), c(nsc,nob-1), r(nsc,nob), r(nsc,nob-
)];
        %Z(nob-2,:)=[inc(nsc,nob),r(nsc,nob)];
    end

%find parameter value estimates

%[b(nsc,:),Qval(nsc)]=fminsearch(@GMMObjFunId,[.75,.5,.25,.2,.15]);
[b(nsc,:),Qval(nsc)]=fminsearch(@GMMObjFunId,[.75,.5,.25,.2]);
%[b(nsc,:),Qval(nsc)]=fminsearch(@GMMObjFunId,[.75,.85]);

    %first time to get pseudo parameters (consistent estimates)

temp=VOM(b(nsc,:))
%temptemp=inv(temp*temp')
%owm=inv(VOM(b(nsc,:))*VOM(b(nsc,:))');

%[b(nsc,:),Qval(nsc)]=fminsearch(@GMMObjFun,[.75,.5,.25,.2,.15]);
    %second time to get more efficient estimates

%find var cov matrix for estimates
vcb(:, :, nsc)=VarCov(b(nsc, :));

end

```

GMMObjFunId.m

```

function gid = GMMObjFunId(y)

gid= VOM(y) '*VOM(y);
%(1x5) (5x1)=(1x1)

% Compute GMMObjFunId(gid) for some y values (to be fimsearched over)
% GMMObjFun value =0 if perfectly specified

%called by Main Code (MCP=metrics cooper proj)

%Take in a vector of moment conditions (VOM) (1x5)
    %calls VOM.m

%GMMObjFunId value(depends on parameter values, to be min)

```

VOM.m

```
function v = VOM(y)
```

```
global Z ob
```

```
v = ((1/(ob-3))*Euler(y)'*Z)';
```

```
%((1x1) (1x9) (9x5))'=(5x1)
```

```
% Compute vector of moments' values at y (to be fminsearched over)
```

```
% interacts euler residuals with instruments
```

```
%called by GMMObjFun.m
```

```
%Take in a vector of euler conditions (9x1)
```

```
    %calls Euler.m
```

```
%Take in a Instruments matrix
```

```
    %call variable Z
```

```
%Output a vector of functions (moments) at parameter values y
```

Euler.m

```
function u = Euler(y)
```

```
global ob nobx r c
```

```
u=zeros(ob-3,1);
```

```
for nobx=3:ob-1
```

```
    u(nobx-2)=BETA(y)*(r(nobx+1)/r(nobx))*(c(nobx+1)/c(nobx))^(1/y(1))-1;
```

```
end
```

```
% Compute Euler Residuals at y (to be fminsearched over)
```

```
%called by VOM.m
```

```
%Take in a vector of Consumption Values (c=10x1)
```

```
    %calls c.mat
```

```
%Take in a vector of interest rates
```

```
    %call variable r.mat
```

```
%Take in vector of beta regressors x
```

```
    %call variable x.mat
```

```
%Output a vector of euler equations (@y)
```

BETA.m

```
function beta=BETA(y)
```

```
%global nobx age age2 famsize nsc
```

```
global nobx age famsize nsc
```

```
%global nobx age age2
```

```
%beta=y(2)+y(3)*age(1,nobx)+y(4)*age2(1,nobx)+y(5)*famsize(nsc,nobx);
```

```
beta=y(2)+y(3)*age(1,nobx)+y(4)*famsize(nsc,nobx);
```

```
%beta=y(2)+y(3)*age(1,nobx)+y(4)*age2(1,nobx);
```

```
%beta=y(2);
```

GMMObjFun.m

```
function g = GMMObjFun(y)
```

```
global owm
```

```
g = VOM(y)'*owm*VOM(y);
```

```
% (1x5) (5x5) (5x1) = (1x1)
```

```
%g= VOM(y)'*OWM(y)*VOM(y);
```

```
% Compute GMMObjFun(g) for some y values (to be fimsearched over)
```

```
% GMMObjFun value =0 if perfectly specified
```

```
%called by Main Code (MCP=metrics cooper proj)
```

```
%Take in a vector of moment conditions (VOM) (1x5)
```

```
    %calls VOM.m
```

```
%Take in a weighting matrix
```

```
    %call OWM.m
```

```
%Output gmm objective function value(depends on parameter values, to be min)
```

VarCov.m

```
function vcv = VarCov(y)
```

```
global owm
```

```
vcv = inv(JAC(y)'*owm*JAC(y));
```

```
% Compute var/cov matrix of parameter estimates y
```

```
%called by MCP.m
```

```
% Take in a vector of consumption values
```

```

% Take in a vector of interest rates
% Take in a matrix of instruments
% Take in parameter estimate
% Take in optimal weighting matrix
% Output a VarCovMatrix

```

JAC.m

```
function jac = JAC(y)
```

```
global Z ob nobj J jac
```

```
J=zeros(4,5);
```

```
%parameters,instruments
```

```
for nobj=1:ob-3
```

```
    jac=kron(JACJ(y)',Z(nobj,:));
```

```
    J=jac+J;
```

```
end
```

```
jac=(1/(ob-3))*J;
```

JACJ.m

```
function jacj = JACJ(y)
```

```
global r c nobj Z
```

```
jacj=zeros(1,4);
```

```
jacj(1,1)=BETA(y)*(r(nobj+2)/r(nobj+1))*(c(nobj+3)/c(nobj+2))^( -
1/(y(1)))*log(c(nobj+3)/c(nobj+2))*inv(y(1)^2);
```

```
jacj(1,2)=BETA(y)*(r(nobj+2)/r(nobj+1))*(c(nobj+3)/c(nobj+2))^( -1/(y(1)))*1;
```

```
for np=3:4
```

```
    jacj(1,np)=BETA(y)*(r(nobj+2)/r(nobj+1))*(c(nobj+3)/c(nobj+2))^( -
1/(y(1)))*Z(nobj,np);
```

```
end
```